2　式の値

10

(1)

$$
\begin{aligned}
与式 &= \left(x^2 + 2xy + y^2\right) - 2xy \\
&= (x+y)^2 - 2xy \\
&= \left(\frac{5}{6}\right)^2 - 2\cdot\frac{1}{6} \\
&= \frac{13}{36}
\end{aligned}
$$

(2)

$$
\begin{aligned}
与式 &= \frac{x^2 + y^2}{xy} \\
&= \frac{\frac{13}{36}}{\frac{1}{6}} \\
&= \frac{13}{6}
\end{aligned}
$$

(3)

$$
\begin{aligned}
与式 &= (x+y)^3 - 3xy(x+y) \\
&= \left(\frac{5}{6}\right)^3 - 3\cdot\frac{1}{6}\cdot\frac{5}{6} \\
&= \frac{35}{216}
\end{aligned}
$$

11

$$
\begin{aligned}
x + y &= \frac{4\left(\sqrt{6}-\sqrt{2}\right) + 4\left(\sqrt{6}+\sqrt{2}\right)}{\left(\sqrt{6}+\sqrt{2}\right)\left(\sqrt{6}-\sqrt{2}\right)} \\
&= \frac{8\sqrt{6}}{6-2} \\
&= 2\sqrt{6}
\end{aligned}
$$

$$
\begin{aligned}
xy &= \frac{16}{\left(\sqrt{6}+\sqrt{2}\right)\left(\sqrt{6}-\sqrt{2}\right)} \\
&= \frac{16}{6-2} \\
&= 4
\end{aligned}
$$

よって，

ア

$$\begin{aligned}\text{与式}&=(x+y)^2-xy\\&=\left(2\sqrt{6}\right)^2-4\\&=24-4\\&=20\end{aligned}$$

イ

$$\begin{aligned}\text{与式}&=\left(x^3+3x^2y+3xy^2+y^3\right)-2x^2y-2xy^2\\&=(x+y)^3-2xy(x+y)\\&=\left(2\sqrt{6}\right)^3-2\cdot4\cdot2\sqrt{6}\\&=48\sqrt{6}-16\sqrt{6}\\&=32\sqrt{6}\end{aligned}$$

12

(1)

$\sqrt{14}=a+b \quad \therefore b=\sqrt{14}-a$ ・・・①

$3^2<14<4^2$ より，$3<\sqrt{14}<4$ ・・・②

②より，$a=3$

よって，①より，$b=\sqrt{14}-3$

(2)

$$\begin{aligned}\frac{1}{b}&=\frac{1}{\sqrt{14}-3}\\&=\frac{\sqrt{14}+3}{\left(\sqrt{14}-3\right)\left(\sqrt{14}+3\right)}\\&=\frac{\sqrt{14}+3}{5}\end{aligned}$$

これと②より，$\dfrac{3+3}{5}<\dfrac{1}{b}<\dfrac{4+3}{5}$ すなわち$1.2<\dfrac{1}{b}<1.4$

これより，$c=1$

$\dfrac{1}{b}=c+d$ だから，

$$\begin{aligned}d&=\frac{1}{b}-c\\&=\frac{\sqrt{14}+3}{5}-1\\&=\frac{\sqrt{14}-2}{5}\end{aligned}$$

13

$\dfrac{x+y}{3}=\dfrac{y+z}{6}=\dfrac{z+x}{7}=k$ とすると，

$x+y=3k$ ・・・① $y+z=6k$ ・・・② $z+x=7k$ ・・・③

①＋②＋③より， $2(x+y+z)=16k$ $\therefore x+y+z=8k$ ・・・④

④－②，④－③，④－①より， $x=2k,\ y=k,\ z=5k$

$$\begin{aligned}\therefore \frac{x^3+y^3+z^3}{xyz}&=\frac{8k^3+k^3+125k^3}{10k^3}\\&=\frac{67}{5}\end{aligned}$$

14

(1)

ア

$$\left(x+\frac{1}{x}\right)^2=28$$

$$\begin{aligned}\left(x+\frac{1}{x}\right)^2&=x^2+2\cdot x\cdot\frac{1}{x}+\left(\frac{1}{x}\right)^2\\&=x^2+2+\frac{1}{x^2}\end{aligned}$$

より，

$$x^2+2+\frac{1}{x^2}=28 \quad \therefore x^2+\frac{1}{x^2}=26$$

イ

$$\begin{aligned}x^3+\frac{1}{x^3}&=\left(x+\frac{1}{x}\right)^3-3\cdot x\cdot\frac{1}{x}\left(x+\frac{1}{x}\right)\\&=\left(2\sqrt{7}\right)^3-3\cdot 2\sqrt{7}\\&=50\sqrt{7}\end{aligned}$$

ウ

$$\begin{aligned}x^4+\frac{1}{x^4}&=\left(x^2+\frac{1}{x^2}\right)^2-2\cdot x^2\cdot\frac{1}{x^2}\\&=26^2-2\\&=674\end{aligned}$$

エ

$$x^5+\frac{1}{x^5}=\left(x^2+\frac{1}{x^2}\right)\left(x^3+\frac{1}{x^3}\right)-\left(x+\frac{1}{x}\right)$$
$$=26\cdot 50\sqrt{7}-2\sqrt{7}$$
$$=1298\sqrt{7}$$

(2)

ア

$a^3+b^3=(a+b)^3-3ab(a+b)$ より， $-19=(-1)^3-3ab\cdot(-1)$ $\quad\therefore ab=-6$

よって，

$$a^2+b^2=(a+b)^2-2ab$$
$$=(-1)^2-2\cdot(-6)$$
$$=13$$

イ

$$a^5+b^5=\left(a^2+b^2\right)\left(a^3+b^3\right)-a^2b^3-a^3b^2$$
$$=\left(a^2+b^2\right)\left(a^3+b^3\right)-(ab)^2(a+b)$$
$$=13\cdot(-19)-(-6)^2\cdot(-1)$$
$$=-211$$

15

$$\sqrt{7+4\sqrt{3}}=\sqrt{7+2\sqrt{12}}$$
$$=\sqrt{\left(\sqrt{4}+\sqrt{3}\right)^2}$$
$$=\sqrt{\left(2+\sqrt{3}\right)^2}$$
$$=2+\sqrt{3}$$

$\sqrt{3}$ の整数部分は 1 だから， $\sqrt{7+4\sqrt{3}}$ すなわち $2+\sqrt{3}$ の整数部分 $a=3$

これと $2+\sqrt{3}=a+b$ より， $b=\sqrt{3}-1$

$$\therefore\frac{a}{b}-\frac{b}{a+b-1}=\frac{3}{\sqrt{3}-1}-\frac{\sqrt{3}-1}{2+\sqrt{3}-1}$$
$$=\frac{3}{\sqrt{3}-1}-\frac{\sqrt{3}-1}{\sqrt{3}+1}$$
$$=\frac{3\left(\sqrt{3}+1\right)-\left(\sqrt{3}-1\right)^2}{\left(\sqrt{3}-1\right)\left(\sqrt{3}+1\right)}$$
$$=\frac{5\sqrt{3}-1}{2}$$

16

ア

$$\alpha^2+\sqrt{3}\beta-\left(\beta^2+\sqrt{3}\alpha\right)=0$$

$$\begin{aligned}\alpha^2+\sqrt{3}\beta-\left(\beta^2+\sqrt{3}\alpha\right)&=\alpha^2-\beta^2-\sqrt{3}(\alpha-\beta)\\&=(\alpha-\beta)(\alpha+\beta)-\sqrt{3}(\alpha-\beta)\\&=(\alpha-\beta)\left(\alpha+\beta-\sqrt{3}\right)\end{aligned}$$

より，$(\alpha-\beta)\left(\alpha+\beta-\sqrt{3}\right)=0$

$\alpha\neq\beta$ だから，$\alpha+\beta-\sqrt{3}=0 \quad \therefore \alpha+\beta=\sqrt{3}$

イ

$$\alpha^2+\sqrt{3}\beta+\left(\beta^2+\sqrt{3}\alpha\right)=2\sqrt{6}$$

$$\begin{aligned}\alpha^2+\sqrt{3}\beta+\left(\beta^2+\sqrt{3}\alpha\right)&=\alpha^2+\beta^2+\sqrt{3}(\alpha+\beta)\\&=(\alpha+\beta)^2-2\alpha\beta+\sqrt{3}(\alpha+\beta)\\&=3-2\alpha\beta+3\\&=2(3-\alpha\beta)\end{aligned}$$

より，$3-\alpha\beta=\sqrt{6} \quad \therefore \alpha\beta=3-\sqrt{6}$

ウ

$$\begin{aligned}\frac{\beta}{\alpha}+\frac{\alpha}{\beta}&=\frac{\alpha^2+\beta^2}{\alpha\beta}\\&=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}\\&=\frac{3-2\left(3-\sqrt{6}\right)}{3-\sqrt{6}}\\&=\frac{-3+2\sqrt{6}}{3-\sqrt{6}}\\&=\frac{\left(-3+2\sqrt{6}\right)\left(3+\sqrt{6}\right)}{\left(3-\sqrt{6}\right)\left(3+\sqrt{6}\right)}\\&=1+\sqrt{6}\end{aligned}$$

17

(1)

$$(x-2)(y-2)(z-2)=xyz-2(xy+yz+zx)+4(x+y+z)-8$$
$$=xyz-xyz+4a-8$$
$$=4a-8$$

(2)

条件より，$(x-2)(y-2)(z-2)=0$ が成り立つから，$4a-8=0 \quad \therefore a=2$

$$x^3+y^3+z^3=x^3+y^3+z^3-3xyz+3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)+3xyz$$
$$=(x+y+z)\{(x+y+z)^2-3(xy+yz+zx)\}+3xyz$$
$$=a\{a^2-3(xy+yz+zx)\}+3xyz$$
$$=2\{4-3(xy+yz+zx)\}+3xyz$$
$$=8-6(xy+yz+zx)+3xyz$$
$$=8+3\{xyz-2(xy+yz+zx)\}$$
$$=8$$